Alternating Finite Automata on Ω Words

Avik Shakahari — Shobhit Singh

Logic Automata Games

May 11, 2024

Alternating finite automata

- → An alternating finite automaton (abbreviated afa) is a sextuple $M = (Q, f, \Sigma, \Delta, q_0, \mathscr{F})$, where:
- → Q is a finite set of states and f is a mapping $f : Q \to \{$ and, or $\}$. If $f(q) =$ and (resp. or), q is called a universal state (resp. existential state).
- → Σ is a finite alphabet.
- → $\Delta(q, a) = \{p \mid (\dot{q}, a, p)$ is in $\Delta\}$ is not empty for any q in Q and any a in Σ.
- → q_0 is initial state.
- → $\mathscr{F}$ is a family of subsets of Q. For F in $\mathscr{F}, F$ is called a final set and elements in F are called final states. If $\mathscr{F}$ consists of a single final set, then we say that M is simple.

Computation Tree

$x = x_1 x_2 x_3 \ldots$ be in Σ^w, where x_n is in Σ for $n \geqslant 1$. A computation tree $T(M, x)$ of M on x is an infinite labelled tree satisfying the following conditions:

- → The nodes are labelled with the elements in Q. In particular, the root of $T(M, x)$ is labelled with q_1.
- → For each $n \geqslant 1$, the edges between level n and level $n + 1$ are labelled with x_n.
- → If node v in level n is labelled with a universal state q, then v has a child labelled with p for each p in $\Delta(q, x_n)$.
- → If node v in level n is labelled with an existential state q, then v has exactly one child labelled with p for some p in $\Delta(q, x_n)$.

Run

An infinite path α in $T(M, x)$ beginning at the root is called a run in $T(M, x)$. For a run α, we define

- $\rightarrow$ $I(\alpha) = \{q \mid \text{state } q \text{ occurs in } \alpha \text{ infinitely many times }\}$,
- $\rightarrow$ $O(\alpha) = \{q \mid \text{state } q \text{ occurs in } \alpha\}$.

Accepting Conditions

- $\rightarrow$ $C_1 : I(\alpha) \cap F \neq \emptyset$
- $\rightarrow$ $C_2 : I(\alpha) \subseteq F$
- $\rightarrow$ $C_3 : O(\alpha) \cap F \neq \emptyset$
- $\rightarrow$ $C_4 : O(\alpha) \subseteq F$

Automata Classes

For $i = 1, \ldots, 4$, we define

1. $\mathscr{A}_i = \{L_i(M) \mid M$ is an *afa*,
2. $\mathscr{A}_i^s = \{L_i(M) \mid M$ is a simple *afa*$\}$,
3. $\mathscr{N}_i = \{L_i(M) \mid M$ is an *nfa*$\}$,
4. $\mathscr{N}_i^s = \{L_i(M) \mid M$ is a simple *nfa*$\}$,
5. $\mathscr{D}_i = \{L_i(M) \mid M$ is a *dfa*$\}$,
6. $\mathscr{D}_i^s = \{L_i M) \mid M$ is a simple *dfa*$\}$.

Automata Classes

	C_1	C_2	C_3	C_4
$\mathscr{D}_i$	G_δ^R	F_σ^R	G^R	F^R
$\mathscr{N}_i$	R	F_σ^R	G^R	F^R
$\mathscr{A}_i$	R	R	G^R	F^R